

# ROUTINE 1

## MISSING NUMBERS

When the numbers are missing from a problem, students must first think about the concept or idea of the situation and then begin the process of seeking the correct numbers to enter into the problem. It is well known that when numbers are presented in a problem, students often ignore the concept and work with only the numbers. This routine pushes students to start with the concept first.

Fill in the blank by finding the largest and smallest integers that will make the quadratic factorable (produce real roots).

$$x^2 + 2x + \square = 0$$

### ABOUT THE ROUTINE

*Missing Numbers* is a routine that uses various high school concepts and representations to develop student thinking and reasoning about numbers, variable hopes, and relationships. In this routine, students consider open problems where some of the numbers are missing and they must enter the numbers to satisfy a given condition. This activity is distinctive because often students identify numbers in a problem

first without thinking about the concept, idea, or theorem. The strength of this routine is that it gives the instructor insight into students' reasoning. Moreover, there is often more than one correct answer, so students can think about many different pathways. This routine is inspired by the work of Marcy Cook and the website <http://www.openmiddle.com/>.

The majority of these problems

- often have multiple ways of solving them as opposed to a problem where you are told to solve it using a specific method,
- involve optimization such that it is easy to get an answer but more challenging to get the best or optimal answer, and
- appear to be simple and procedural in nature but turn out to be more challenging and complex when you start to solve it.

### WHY IT MATTERS

This routine helps students and highlights Mathematical Practice 3 (construct viable arguments and critique the reasoning of others) and Mathematical Practice 7 (look for and make use of structure). It also helps students

- consider the conceptual understanding first,
- look for and manipulate patterns and structure within relationships,



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- develop confidence with conceptual understanding and computation,
- communicate their reasoning with others, and
- listen actively to the reasoning of others.

## WHAT THEY SHOULD UNDERSTAND FIRST

Students must first understand the conceptual underpinning of the example. The content of the examples draws from algebra, geometry, and algebra II. Sufficient content knowledge of the given is necessary for students to explore the problems.

## WHAT TO DO

1. Draw or project the problem and the given constraints for the missing numbers.
2. Give students a few minutes to use various strategies (guess/check, graphing calculators, or known theorems) to solve for the missing numbers.
3. Students can be encouraged to use any tools they usually have access to (textbook, notes, calculators, computers, people, etc.).
4. Have students share their results with classmates in small groups or with partners.
5. Bring the class together. Ask students to share their responses.
6. After student ideas are collected, discuss the solutions offered by the class.
7. Honor and explore both accurate and flawed reasoning.
8. Explore additional values if time permits.

## ANTICIPATED STRATEGIES FOR THIS EXAMPLE

In this example, many students will initially realize that when filling in the blank with 1, the expression is factorable as  $(x+1)^2$ . And when  $x > 1$ , the expression has no real roots. For all values of  $x < 1$ , the expression will always have two real roots. This could also be seen by graphing the expression and using a slider for the value of the missing number. Moreover, students could examine the discriminant and determine when it is greater than or equal to zero.

Fill in the blank by finding the largest and smallest integers that will make the quadratic factorable (produce real roots).

$$x^2 + 2x + \square = 0$$

## ADDITIONAL EXAMPLES

## ALGEBRA

This first group of examples (1–14) highlights problem solving and reasoning with algebra. In many of the examples, it is not necessary to actually solve for  $x$  but rather to reason or use number sense to determine the sign of the variable. If students use guess/check to find numbers that “work,” you might ask students to find another solution or generalize the solution.

- 1 Insert any numbers between 1 and 9 only to make the equation have an integer solution  $> 0$ .

$$\square x + \square = \square x + \square$$

1. Answers will vary here, such as

$$3x + 4 = 2x + 5 \text{ and thus } x = 1$$

Generally, if we have  $ax + b = cx + d$  and want  $x > 0$ , then one possible way is to let  $a > c$  and  $b < d$ .

- 2 Insert any numbers between 1 and 9 only to make the equation have NO solution.

$$\square x + \square = \square x + \square$$

2. To have no solution, both coefficients of  $x$  must be the same number and the other two blanks must be different numbers. Here is an example:  $3x + 7 = 3x + 4$

- 3 Insert only the numbers between 1 and 4 to create an equation with a solution as close as possible to 0.

$$\square x + \square = \square x + \square$$

3. The 24 possible ways to do this are:

$x + 2 = 3x + 4$	$2x + 1 = 3x + 4$
$x + 2 = 4x + 3$	$2x + 1 = 4x + 3$
$x + 3 = 2x + 4$	$2x + 3 = x + 4$
$x + 3 = 4x + 2$	$2x + 3 = 4x + 1$
$x + 4 = 2x + 3$	$2x + 4 = x + 3$
$x + 4 = 3x + 2$	$2x + 4 = 3x + 1$
$3x + 1 = 2x + 4$	$4x + 1 = 2x + 3$
$3x + 1 = 4x + 2$	$4x + 1 = 3x + 2$
$3x + 2 = x + 4$	$4x + 2 = x + 3$
$3x + 2 = 4x + 1$	$4x + 2 = 3x + 1$
$3x + 4 = x + 2$	$4x + 3 = x + 2$
$3x + 4 = 2x + 1$	$4x + 3 = 2x + 1$

- 4 Insert numbers between 1 and 9 only to create an equation with a solution as close as possible to 0.

$$\square x + \square = \square x + \square$$

4. There are many equations that will get to the correct answer of  $1/8$  or  $-1/8$ . Here are some examples:

$$9x + 3 = 1x + 4$$

$$1x + 7 = 9x + 8$$

## GEOMETRY

In this set of examples (25–30), students might use graph paper, a graphing calculator, or the desmos app to explore multiple solutions on the coordinate plane. Examples 29 and 30 highlight triangle sum and volume of a cylinder.

**25** Fill in the boxes (ordered pairs) with numbers between 1 and 9 to create a right triangle ABC.

A (□, □)

B (□, □)

C (□, □)

**25.** Multiple solutions are possible including these:

(1, 3)(2, 6)(8, 4)

(1, 7)(2, 8)(4, 6)

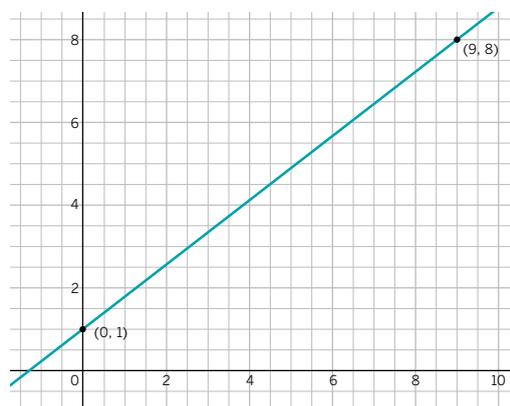
(1, 6)(3, 8)(4, 7)

**26** Fill in the boxes (ordered pairs) with numbers between 1 and 9 to create a line segment with greater possible length.

A (□, □)

B (□, □)

**26.**



One possible solution is A (0, 1) and B (9, 8) with the distance of  $AB = 11.4$ .

## NOTES

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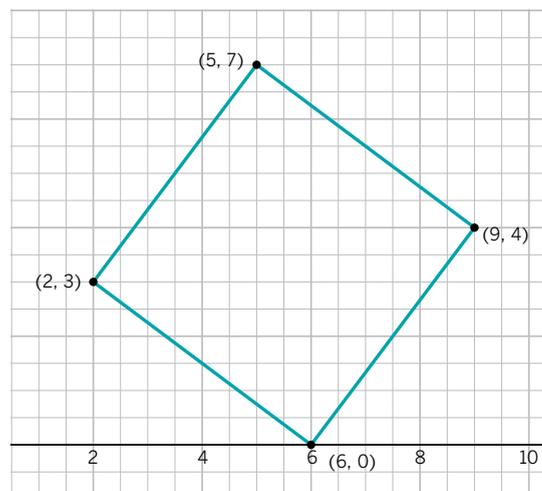
- 27 One vertex of a square is at  $(6, 0)$ . Fill in the boxes (ordered pairs) with numbers between 1 and 9 to complete the square.

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27. One possible solution is shown below.

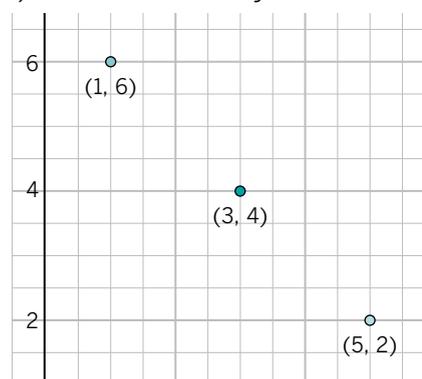


- 28 Fill in the blanks with the numbers 1 to 9 to create two points equidistant from  $(3, 4)$ .

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28. One possible solution is shown below. Both points are  $\sqrt{8}$  away from  $(3, 4)$ . This could also be seen by drawing slope triangles and displaying that each point is 2 away from  $(3, 4)$  in both the  $x$  and  $y$  distance.



## NOTES

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