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Preface

Mathematics Assessment Probes

OVERVIEW

Formative assessment informs instruction and supports learning through varying methods and strategies aimed at determining students' prior knowledge of a learning target and using that information to drive instruction that supports each student in moving toward understanding of the learning target. Questioning, observation, and student self-assessment are examples of instructional strategies educators can incorporate to gain insight into student understanding. These instructional strategies become *formative assessments* if the results are used to plan and implement learning activities designed specifically to address the specific needs of the students.

This book focuses on using short sets of diagnostic questions, called Mathematics Assessment Probes. The Probes are designed to elicit prior understandings and commonly held misunderstandings and misconceptions. This elicitation allows the educator to make sound instructional choices, targeted at a specific mathematics concept and responsive to the specific needs of a particular group of students.

Diagnostic assessment is as important to teaching as a physical exam is to prescribing an appropriate medical regimen. At the outset of any unit of study, certain students are likely to have already mastered some of the skills that the teacher is about to introduce, and others may already understand key concepts. Some students are likely to be deficient in prerequisite skills or harbor misconceptions. Armed with this diagnostic information, a teacher gains greater insight into what to teach. (McTighe & O'Connor, 2005)

The Mathematics Assessment Probes provided here are tools that enable high school teachers to gather important insights in a practical way and that provide immediate information for planning purposes.

AUDIENCE

The first collection of Mathematics Assessment Probes and the accompanying Teacher Notes were written for the busy classroom teacher eager for thoughtful, research-based diagnostic assessments focused on learning difficulties and aimed at enhancing the effectiveness of mathematics instruction. Since the publication of the first three *Uncovering Student Thinking in Mathematics* resources (Rose & Arline, 2009; Rose, Minton, & Arline, 2007; Rose Tobey & Minton, 2011), we have continually received requests for additional probes. Both teachers and education leaders have communicated the need for a collection of research-based probes that focus on a narrower grade span. In addition to additional probes for each grade span, educators were eager for an alignment of the probes to the Common Core Mathematics Standards (Council of Chief State School Officers, 2010). In response to these requests, we set to work writing, piloting, and field testing a more extensive set of probes for high school teachers with a focus on targeting mathematics concepts within the new standards. This book is one in a series of *Uncovering* resources for the K–2, 3–5, 6–8, and 9–12 grade spans.

ORGANIZATION

This book is organized to provide readers with the purpose, structure, and development of the Mathematics Assessment Probes as well as to support the use of applicable research and instructional strategies in mathematics classrooms.

Chapter 1 provides in-depth information about the process and design of the Mathematics Assessment Probes along with the development of an action-research structure we refer to as a QUEST Cycle. Chapters 2 through 6 contain the collection of probes categorized by concept strands with accompanying Teacher Notes to provide the specific research and instructional strategies for addressing students' challenges with mathematics. Chapter 7 highlights instructional considerations and images from practice to illuminate how easily and in how many varied ways the probes can be used in mathematics classrooms. This chapter also highlights how use of the probes can support students' proficiency with the Common Core's Mathematical Practices.