The total time for the wind round-trip is

$$t = \frac{d}{350} + \frac{d}{250}.$$

The "total rate of speed" or rate for the wind round-trip (which is really the average rate of speed) is

$$r = \frac{2d}{\frac{d}{250} + \frac{d}{350}} = \frac{2(350)(250)}{250 + 350}, \text{ so}$$

$$r \approx 291.67,$$

which is slower than the no-wind trip, and hence took more time.

This average of rates (≈ 291.67) is called the *harmonic mean* between the two speeds of 350 mph and 250 mph. It might prove fruitful to digress to a discussion of the harmonic mean and its related series, the harmonic sequence.[1]

The harmonic mean for a and b is $\frac{2ab}{a+b}$, and for three numbers, *a, b,* and *c,* the harmonic mean is $\frac{3abc}{ab + bc + ac}$.

It is important to include as part of any good mathematics instruction program that the averaging of rates cannot be handled as though they were ordinary numbers, since they exist over varying amounts of time.

49. WHY DOES 0.99999 . . . = 1?

In mathematics, the repeating decimal 0.99999. . . (sometimes denoted as $.\overline{9}$) is a real number and can be proved to be the number 1. Various proofs have been developed; however, these proofs may have varying contextual suppositions, which include historical perspectives and cognitive levels of the student.

In order to show why 0.99999 . . . = 1, we will use the method of converting a repeating decimal into fraction form. Keep in mind that this is an intuitive approach, which informally proves the conjecture. Before we begin, let us consider the following analogous example:

To convert .2525 . . . into fraction form, we will let $x = .\overline{25}$ and then multiply both sides of the equation by 100, to get $100x = 25.252525 \ldots = 25.\overline{25}$. By subtracting $x = .\overline{25}$ from both sides of the equation, we get $99x = 25$: thus $x = \frac{25}{99}$.

Using the same method as above, let us now consider 0.99999. . . $= .\overline{9}$. We let $x = .\overline{99}$, and then multiply both sides of the equation by 100 to get $100x = 99.\overline{99}$. Subtract $x = .\overline{99}$ from both sides of the equation, to get $99x = 99$; thus $x = \frac{99}{99}$, which is equal to 1.

We can also look at this with the following example:

Suppose we wish to find the sum $0.\overline{6} + 0.\overline{3}$.

We can do this either as decimals or as fractions:

As decimals, $0.\overline{6} + 0.\overline{3} = 0.\overline{9}$.

As fractions, $0.\overline{6} + 0.\overline{3} = \frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$.

This would then imply that $0.\overline{9} = 1$.

50. IS A ROAD WITH A SLOPE OF 20% TWICE AS STEEP AS A ROAD WITH A 10% SLOPE?

This might seem like a silly question, but a thorough understanding of this problem helps to avoid basic misunderstandings later on.

What does it mean for a road to be twice as steep as another one? The answer of course depends on how you measure "steepness." In school we have two such possibilities: We could either use the grade or slope defined as the quotient of rise over run, that is, by dividing the vertical gain (measured on the y-axis) by the horizontal distance (measured on the x-axis). Or we could quantify steepness by the angle of inclination, that is, the angle of the road with the horizontal direction. Students with a knowledge of trigonometry could already convert one measure into the other by using the tangent function on their pocket calculator.

Figure 3.2

Source: © Getty Images/Thinkstock

Figure 3.3 shows that twice as steep in angles is quite different from twice as steep in slope, except for very small inclination angles α, where $\tan\alpha \approx \alpha$.