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Foreword

> *What kind of a day was today? A day like any other day, filled with those events that alter and illuminate our times. And you are there.*
>
> —Walter Cronkite

Today some educators and politicians in the United States are salivating at the prospect of national K–12 curricula in mathematics. Is this the culmination of the so-called Math Wars? The Common Core School Standards in Mathematics (CCSS), written by a handful of people and hastily reviewed and adopted by states, will now require an intense rearguard action by those of us who care about what the curriculum really is and how it actually gets implemented in the millions of classrooms around our nation. The CCSS is long on content and comparatively silent on process.

Does anyone remember why the National Council of Teachers of Mathematics (NCTM) developed their initial standards for curriculum in 1989, revised them in 2000 with extensive treatment of both content and process, and then developed preK–8 grade-level-specific Focal Points for the critically important concepts in 2006?

Think back to the Third International Mathematics and Science Study, especially the videotaped component that by 2003 had analyzed thousands of hours of 100 eighth-grade teachers in each of seven high-performing countries as compared to 100 U.S. teachers. The researchers discovered several things:

- Each country had its own particular "culture of teaching mathematics."
- Teachers in each of the other seven countries had a typical way of helping students grapple with conceptually rich math problems through some form of active questioning and dialogue. There were characteristic patterns of engaging students' thinking and making connections among the concepts of the problem.

- However, none of the 100 U.S. math teachers exhibited any such behavior. Instead, they told the students what procedures to use to get the right answer, turning the rich, conceptual task into a plug and chug, computational exercise. In fact, a third of the time, they merely gave them the answer!!!

Is it any wonder that so many of our U.S. students believe that math is a hodge-podge of rules to memorize, procedures that one simply *does without thinking?* I am not chastising our math teachers; they are teaching the way they were taught.

The culture of math teaching in the United States includes the following patterns of behavior:

- Showing students what procedure to use to get the right answer rather than helping them understand underlying concepts.
- Suggesting that students look for the key words (e.g., "altogether" to cue them to add the numbers, or "difference" to mean that they should subtract). The teachers do not realize that the message kids get is, "don't bother reading the problem or thinking about what is going on."
- Using mnemonic tricks to help memorize procedures, for example, "Please Excuse My Dear Aunt Sally" where PEMDAS signifies the order of operations (parentheses, exponents, multiply, divide, add, subtract). Students thus focus on the order rather than the *meaning* of the operations.

Perhaps the epitome of our U.S. culture of teaching math was shared with me by a sixth grader. When confronted with the exercise of dividing a fraction by a fraction, he stated, "Ours is not to reason why, we just invert and multiply!" Alfred Lloyd Tennyson, roll over in your grave! With our God-given capability of abstract thought and reasoning, are we not better served by asking *why*?

In Algebra class, how many denominators must be mindlessly "rationalized"?

How many polynomials must be factored without reference to a context?

Or "Here is an equation, young fellow. Make a table of values for it. Then graph it. Okay, you are finished." "What? You want to know what this is an equation OF?" "Why, it could be many different things, sputter, sputter. Don't ask such ridiculous questions."

Where do we start?

I talk to a lot of parents and teachers each year. I tell them that there are three ideas one must entertain:

1. Arithmetic is not synonymous with mathematics. It is part of math, one of its many branches. *Mathematics is the science of patterns*. There are many wonderful patterns in mathematics that even young children can appreciate.

2. The goal of teaching mathematics is to understand concepts, not to memorize procedures. To teach mathematics for conceptual understanding, teachers must *use principles from cognitive psychology* to help students learn how to think. By their very nature, concepts organize information and help students discern patterns. Concepts in mathematics are abstract relationships that are understood by wrestling with lots of examples. When someone else (teacher, parent, or older sibling) merely tells a child what to do or shows him or her how to do it, the child is denied the experience of *thinking through* what is going on here. The child may remember the procedure but not know when to use it appropriately.

3. Most humans, most of the time, think with language. Reading, writing, speaking, and listening are integral to doing thoughtful mathematics. The teacher must have a *dynamic dialogue* with the students, discussing, debating, and thinking about how they are conceiving of the mathematical tasks they are doing. They must be able to read mathematics texts and story problems with full comprehension. They need time to think and write about their conceptions and strategies.

Like the 1001 Arabian Nights of Scheherazade, Margie Pearse and K. M. Walton have written a book that gives teachers 1001 suggestions of where to start. They have organized these suggestions around nine *critical thinking habits,* which will be familiar to educators who have studied literacy and developed a love for language and literature of all genres. These nine critical thinking habits are cognitive processes—habits of mind, thought, and imagination. They encompass reading comprehension strategies, and the authors show how these can help students comprehend mathematics. They include the five fundamental processes of doing mathematics advocated by NCTM (problem solving, reasoning and proving, making connections, communicating one's conceptions, and creating representations). The authors illustrate how to use metaphors and analogies (metaphorical and analogical thinking) to reach even the recalcitrant math student . . . like my daughter.

My daughter, Alicia, was doing fine in mathematics through eighth grade but was becoming increasingly indifferent to it each year. The performing arts were her passion. She excelled in language and literature, but acting, singing, dancing, writing, and directing (which she did in high school and college) were her raison d'être. In college she was required to take one math course. The most basic one allowed was Finite Mathematics. Before she panicked, she found out that she could take Finite Math at the local community college when she was home in the summer (and I could help her). Before she registered, I went to the college bookstore to see what texts were being used by the five different instructors. She signed up for the fellow who was using a text that provided relatively good contexts, offering a modicum of motivation.

The course contained a fair amount of probability. During the first week, we got out a deck of cards to explore poker hands (Critical Thinking Habit 2: Develop Schema and Activate Background Knowledge). A dim recollection stirred. We looked at the first problem: What is the probability of a heart flush? I turned to look at Alicia sitting next to me. She was totally spaced out. “Earth to Alicia. Come in, please.”

“Heart flush. Heart flush,” she muttered. “What a great title for a poem!” Whereupon, she wrote a haiku by that title.

I dragged her back to the poker game, complete with manipulatives. “Okay, there are 52 cards, and 13 of them are hearts. So the probability of the first card being a heart is 13 out of 52 or ¼. The probability that a second card would also be a heart would be 12 out of 51.” And so on. We walked and talked through the problem. “So figure out the probability of all five cards in the hand being hearts. What would it be?”

Her response was immediate, guileless, matter of fact: “Oh, I don’t know. One in a million.”

She thinks in metaphors. So do most humans. Try to go an entire day without using a metaphor.

Margie Pearse and K. M. Walton have spent a decade devouring the research on best practice in teaching mathematics and testing out this work in classrooms. Much of this research base was ignored by the presidential panel in 2008 that was charged with reporting on teaching and learning in math. The panel examined only studies that met the standards of experimental or quasi-experimental research (e.g., the random assignment of students to treatment and control groups). Excellent qualitative research studies, which also include appropriate statistics, describing and analyzing *what* students learn as well as *how* they learn, were readily available. They would have provided the panel with a powerful foundation. Instead, with one swipe of a hand, any research that might have been inspired by Piaget (e.g., clinical observation) was dismissed. Similarly ignored was the landmark synthesis of research on how students learn mathematics sponsored by the National Research Council (2005).

Fortunately, Margie Pearse and K. M. Walton have extensively referenced the practices that they explored, and they supply solid documentation for their suggestions. They have chosen *numeracy* as their flagship concept, and it’s a grand one. Their intent is to draw attention to the fundamental building blocks of mathematical meaningfulness that children must develop to construct higher-level mathematics. In *Early Numeracy,* Wright, Martland, and Stafford (2003) carefully establish the validity of their Learning Framework in Number (LFIN) model, the basis of their Math Recovery program of assessment, intervention, and teaching. They show that the development of initial numeracy by young children ages 4 to 9 years old is absolutely critical to later mathematical success. The practices, activities, and problems Margie Pearse and K. M. Walton elaborate on can provide a mathematically rich environment in which numeracy can flourish.

—*Arthur Hyde*